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Math 310
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1. To convert this repeating decimal into a rational number, we take advantage of the fact that the decimal is repeating. We will take $x = 2.10012\overline{097}$. If we multiply x by 1000, we get $1000x = 2100.12\overline{097}$, or, more transparent of purpose, $1000x = 2100.12097\overline{097}$- as the final decimal series is infinitely repeating, we can pull one iteration out and still have the same number. When x is subtracted from $1000x$, though, we end up subtracting the decimal expansion from itself, and it goes away, leaving us with $999x = 2098.02085$. To get a proper rational number, the numerator and denominator must be integers, so we multiply both sides by 100000 and then divide by 99900000, getting $x = \frac{209802085}{99900000}$.

2. The statement:
$$a_0.a_1a_2 \cdots a_n\overline{0} = a_0.a_1a_2 \cdots (a_n - 1)\overline{9} \tag{1}$$
is equivalent to:
$$a_0.a_1a_2 \cdots a_n = a_0.a_1a_2 \cdots (a_n - 1) + \sum_{i=n+1}^{\infty} \frac{9}{10^i} \tag{2}$$
The limit of the sum above is $10 \cdot 10^{-n+1}$, giving us a_n instead of $a_n - 1$, and making the LH and RH sides of the equation the same. QED.

3. Sorry if I missed this correction in class, but I am assuming you mean to remind us that a set is uncountable iff it's neither finite nor *countably* infinite.
To prove this, we simply restate Cantor's Diagonal Argument. As a set S being countable relies upon a bijection $\mathbb{Z}^+ \to S$, we can prove a set not countable by proving this false. Given that every irrational number has an associated infinite decimal expansion, and this expansion is not $..\overline{9}$ or $..\overline{0}$ as these are equal and finite (see above), for each irrational number, we have a decimal expansion $b_n = a_1a_2a_3 \cdots a_{k-1}.a_ka_{k+1}a_{k+2} \cdots$ where $a_n \in \mathbb{N}$ and $0 \leq a_n \leq 9$. If we were to align the infinite decimal expansions of all the irrationals in a matrix, and created a function $f : \mathbb{Z}^+ \to S$ which mapped the natural numbers to the rows of the matrix, we would have completed a supertask and stunned philosophers around the globe, but we would still be able to construct another infinite expansion b whose decimal expansion was $b = 0.b_1b_2b_3 \cdots b_n \cdots$ where a decimal b_n is 0 when $a_{nn} = 1$ and 1 when not, thus differing from each number in the matrix in at least one decimal. This number b would clearly be a unique number from all the numbers in this matrix which f maps onto, and thus not being in the image of f, making f not surjective, forming a contradiction, and thus, being not countable. Q.E.D.

4. *Determine the cardinality of the following sets:*
   (a) $|\mathbb{R} - \mathbb{Q}| = |\mathbb{R}|$
   (b) $|\mathbb{Q}^{\geq}| = \aleph_0$:
   (c) Where $S = \{\sqrt{n} \mid n \in \mathbb{Q}^{\geq}\}$, $|S| = \aleph_0$
   (d) Where $R = \{ \sqrt[m]{n} \mid n \in \mathbb{N}, m \in \mathbb{N}$, $|R| = \aleph_0$
   (e) Where $A = \{n \in \mathbb{Z} \mid 0 \leq n \leq 41\}$, $|A| = 42$
   (f) The complex numbers: $\mathbb{C} = \{x + iy \mid x, y \in \mathbb{R}\}(i = \sqrt{-1})$: $|\mathbb{C}| = |\mathbb{R}| = 2^{\aleph_0}$

5. (a) $|M| = 707$, where $M = \{a_1, a_2, a_3, \cdots, a_707 \mid a_n \in \mathbb{R}\}$
   (b) $|\emptyset| = 0$
   (c) $|\mathcal{P}(\mathbb{N})| = \aleph_1$
   (d) $|\mathcal{P}(\mathcal{P}(\mathcal{P}(\mathcal{P}(\mathcal{P}(\mathbb{N})))))| = \aleph_5$