Peter Woodman
0127119
Math 310
A. Nichifor

1. *Does the Trichotomy law hold for sets?*
   No, the Trichotomy law does not hold for sets. This can be proven false by the counterexample where set $A = \{a\}$ and set $B = \{b\}$. In this case, $A \not\subset B$ as $a \notin B$, $A \not\supset B$ as $b \notin A$, and clearly $A \neq B$. Q.E.D.

2. *Let $A = \{1, 2, 3, 4\}$. List all elements of $\wp(A)$. List all elements of $A \times A$.*
   If $A = \{1, 2, 3, 4\}$, then
   $\wp(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, A\}$.
   $A \times A = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)\}$

3. *Suppose $A \subseteq B$. Prove that $\wp(A) \subseteq \wp(B)$.*
   Since we know that $A \subseteq B$, we know that the set B contains all the elements in the set A. Given this fact, and given that the power set $\wp(X)$ is defined as the set containing all the possible combinations of the elements in X, it is clear that $\wp(B)$ must contain all possible combinations of the elements in A as it contains all such elements, hence $\wp(B) \supseteq \wp(A)$. Q.E.D.

4. *Prove that if $A \times B = A \times C$ and $A \neq \emptyset$, then $B = C$.*
   Assume that $B \neq C$. This means that there is an element m which exists in $B \cup C$ but is not common to both sets. Assuming it is in B (it does not matter whether the element is in B or C, however), this would mean that $A \times B$ would necessarily include the elements (a, m) for all $a \in A$. These elements would necessarily not be in $A \times C$, as m does not exist in C, and thus, $A \times B \neq A \times C$. Thus, the assumption fails, and we are left with $B = C$. Q.E.D.

5. *Prove that $A \cup (A \cap B) = A$.*
   The union of a set with any subset of itself is simply the original set. The intersection of a set and another set is necessarily a subset of both of the initial sets. Hence, $A \cap B \subseteq A$, and hence, $A \cup (A \cap B) = A$. Q.E.D.

6. (a) *Prove that $\forall x \in \Re, \exists y \in \Re, x + y > 0$.*
   Observe that for any number n in $\Re$, there exists a number $n-1$. Observe also that $n-(n-1) = n+(-n+1) = n-n+1 = 1$. Observe also that $1 > 0$. Therefore, for any x, we can construct a y where $x + y = 1 > 0$. Q.E.D.

   (b) *Prove that $\forall x \in \Re, \exists y \in \Re, xy > 0$*
   This is false, as we find when we examine the case $x = 0$. Zero times anything is zero, so $0 \cdot y \not> 0$.

   (c) *Prove that $\forall x \in \Re, \exists y \in \Re, xy \geq 0$*
   Any number times zero is zero, so for any x, we have at least the solution $y = 0$, so this is a true statement. Q.E.D.

7. The positive: $\exists x \in A, \forall y \in A, x > y$. The negative: $\nexists x \in A, \forall y \in A, x > y$.
   A Case Satisfying The Positive: $A = \{1, 2, 3\}$. A Case Satisfying The Negative: $A = \emptyset$.

8. (a) $f_2(x) = x^3$
   This function is bijective- since $y = f_2(x) \Leftrightarrow y = x^3 \Leftrightarrow x = \sqrt[3]{x}$, for $x, y \in \Re$ we see that there for each element y of $\Re$ has precisely one pre-image under f_2, since, unlike the square root function, the cube root function is defined in all of $\Re$. Hence, this function is invertible. Its inverse is $\sqrt[3]{x}$.

   (b) $f_3(x) = x^3 - x$
   This function is not surjective- we have the counterexample of $y = f_3(1) = 0 = f_3(-1)$. Nor is it injective- the above counterexample proves this as well, as $1 \neq -1$.

   (c) $f_5(x) = e^x$