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Amath 352 - B.Bale
Homework 3

1. (a) $p_1 + p_2 = 1.505$, $p_1 p_2 = 0.1290$
   (b) $p_1 + p_2 = 31.442$, $p_1 p_2 = .85392$

2. (a) The Taylor series expansion to three terms of this function comes out to be $1+x2$. $\int 1 + x^2 = x + \frac{x^3}{3}$, so $\int_0^{1/4} \hat{p} = \frac{1}{4} + \frac{\frac{1}{4}^3}{3} = 0.25520$.
   (b) Three, as the fourth significant figure in the approximation is incorrect.
   (c) Truncation error.

3. (a) We can arrive at this function by using second-order Taylor series expansions of a function $f(x)$. We use h to represent our small step size. If we are to look at the Taylor series expansion of $f(x+h)$, we get

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2} f''(x) + \frac{h^3}{3} f'''(x) + \cdots \tag{1}$$

   For $f(x-h)$, we get

$$f(x-h) = f(x) - hf'(x) + \frac{h^2}{2} f''(x) - \frac{h^3}{3} f'''(x) + \cdots \tag{2}$$

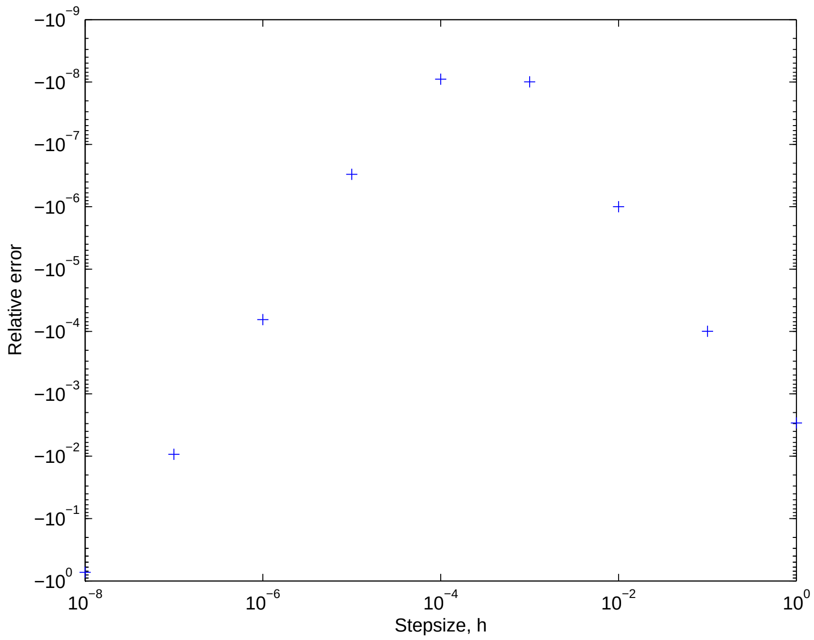
   We now note that if we are to add these two functions together, we get the requested difference function, plus some error from the remaining terms of the Taylor series

$$f(x-h) + f(x+h) = 2f(x) + h^2 f''(x) + \mathcal{O}(h^2) \tag{3a}$$

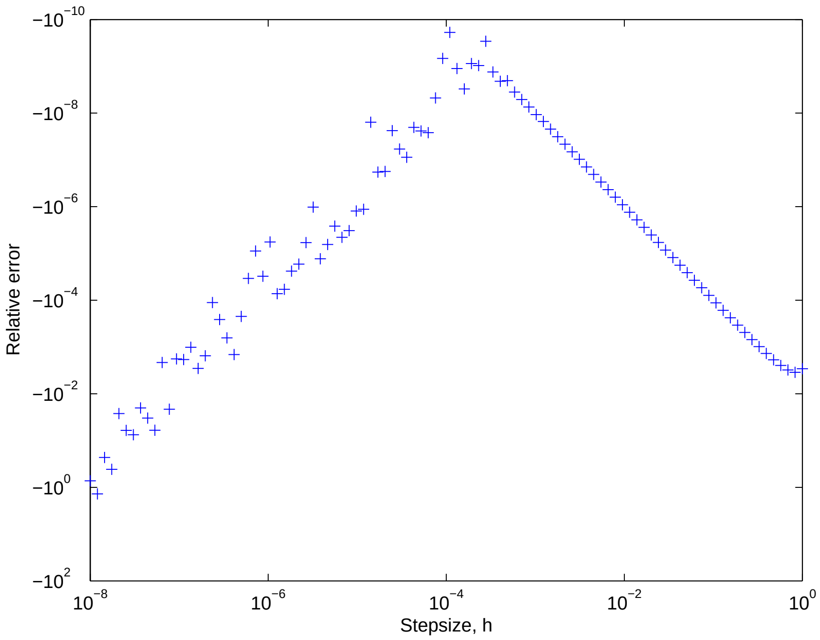
$$f''(x) \equiv \frac{f(x-h) - 2f(x) + f(x+h)}{h^2} + \mathcal{O}(h^2) \tag{3b}$$

   Manipulate this, and we are left with the difference equation, plus the remaining terms of the Taylor series.
   (b) The error is $\mathcal{O}(h^2)$.
   (c) Done. See the back sheets for code.
   (d) Well, given the choice of making a bunch of silly `.m` files for f and f'' or coding the function in (as silly Matlab does not have any sort of lambda function concept), I chose the latter..
   (e) Well, I'm not quite sure about this one, honestly. I believed the error to be $\mathcal{O}(h^2)$, but it may not be this simple. I'll simply start including the program output here.

```
>> fancy(2)
   h    fpp      fppfd       Erel
   1.0e+00  -5.14777  -5.16283    -2.93e-03
   1.0e-01  -5.14777  -5.14828    -9.90e-05
   1.0e-02  -5.14777  -5.14777    -9.97e-07
   1.0e-03  -5.14777  -5.14777    -9.95e-09
   1.0e-04  -5.14777  -5.14777    -9.03e-09
   1.0e-05  -5.14777  -5.14777    -3.02e-07
   1.0e-06  -5.14777  -5.14744    -6.44e-05
   1.0e-07  -5.14777  -5.19584    -9.34e-03
   1.0e-08  -5.14777  -8.88178    -7.25e-01
```

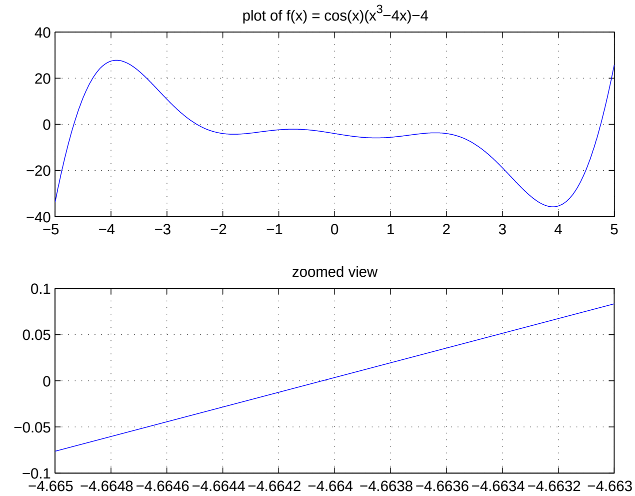


Initially, this graph looked very wrong to me. I verified my code to make sure there wasn't something going wonky in here, and there seemed to be nothing. I tried plotting the function over a smaller step size:

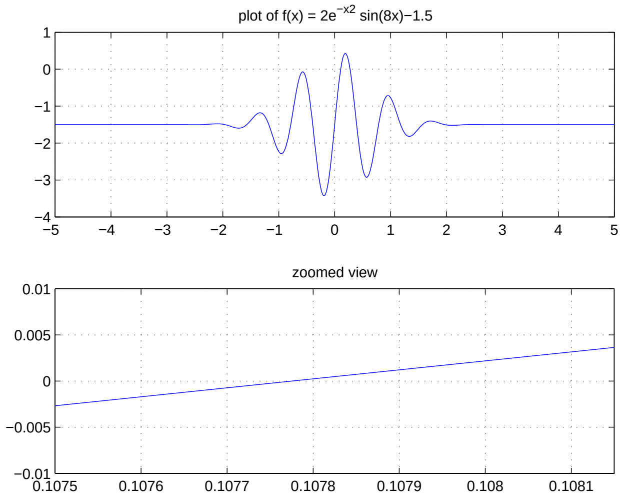


Looks like this approximation just starts going wonky at around $h = 10^{-4}$. I've got two theories. One- matlab problem with rounding. Second- truncation error, in that we are really only using the first three terms of the Taylor series to compute this. Personally, I'm going to go with the matlab problem, as it doesn't seem to be any sort of tipping point- $\mathcal{O}(h^2)$ behavior is observed up to the point where things go wonky, and then they just indeed go straight up crazy.

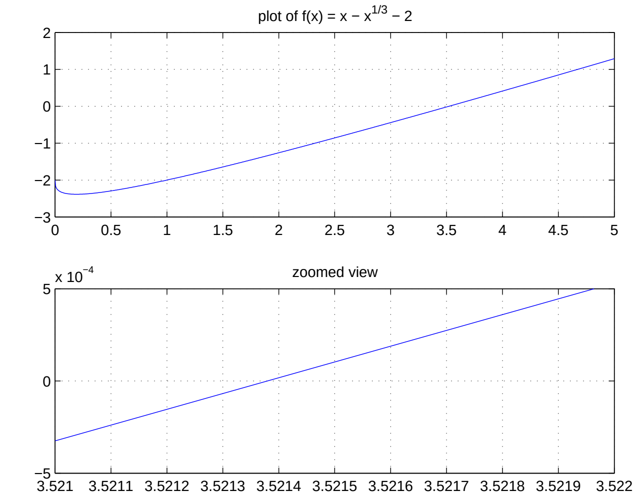
4. (a) Script modified as requested. The root I chose appears to be right around -4.6643.



(b) In this case, it appears there is a root around 0.10778.



(c) The root appears to be around 3.5214



Appendix: Code

-fancy.m-

```
% fancy.m
function [] = fancy(x)

f = x.^2.*sin(x);
fpp = 4*x*cos(x) + 2*sin(x) - (x^2)*sin(x);

h = logspace(0,-8,100)';

fppfd = ((x+h).^2.*sin(x+h) - 2*(x^2*sin(x)) + (x-h).^2.*sin(x-h))./(h.^2);
Erel = abs(fppfd - fpp)./fpp;

fprintf('h  fpp     fppfd       Erel\n');
for k=1:length(h)
    fprintf('%10.1e %9.5f %9.5f %12.2e\n', h(k),fpp, fppfd(k), Erel(k));
end

loglog(h, Erel, '+')
xlabel('Stepsize, h'); ylabel('Relative error');
```

-plotfunction1.m-

```
% plotfunction1.m
%
x = linspace(0,5,1000);   % the domain interval.
% y = cos(x) .* (x.^3 - 4*x) - 4; % the function we are looking at in 4a.
% y = 2 .* exp(-x.^2) .* sin(8.*x) - 1.5 ; % the function we are looking at in 4b.
y = x - x .^ (1/3) -2; % the function we are looking at in 4c.
%
subplot(2,1,1)  % the top plot
plot(x,y)
grid on  % turn the grid on.
% title('plot of f(x) = cos(x)(x^3-4x)-4')          % 4a
% title('plot of f(x) = 2e^{-x}^{2} sin(8x)-1.5')   % 4b
title('plot of f(x) = x - x^{1/3} - 2')             % 4c
%
subplot(2,1,2)  % the bottom plot
plot(x,y)
grid on
% axis([-4.665 -4.663 -.1 .1])              % 4a
% axis([0.1075 0.10815 -.01 .01])           % 4b
axis([3.521 3.522 -.0005 .0005])            % 4c
title('zoomed view')
```