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Amath 352 - B. Bale
Take-home Final

WARNING: ALL BELOW ANSWERS ARE COMPLETE SHIT (I ran out of time after spending 2 hours tracking down a lowercase 'i'. Have some mercy on me.)

1. (a) The matlab function is below. *-hilbert.m-*

```
function [hil,col] = hilbert(n)

hil = zeros(n,n);
for i=1:n
  for j=1:n
    hil(i,j) = 1/(i+j-1);
  end
end

col = ones(n,1);
```

   (b) At around $n = 10$, we start to see some error on the scale of 10^-4. Again, a matlab function to find $\hat{x}$ for a given n-

```
function xapp = xapprox(n)

[H,x] = hilbert(n);
b = H*x;
xapp = H\b;
```

   (c)

```
n = 20;
errnorms = zeros(n,1);
resnorms = zeros(n,1);
for i = 1:n;
  [H,x] = hilbert(i);
  xhat = xapprox(i);
  b = H*x;
  resnorms(i) = norm(b-H*xhat,Inf); errnorms(i) = norm(x - xhat,Inf);
end
```

   After this computation runs, we check errnorms, and find that the thirteenth element is

2. (a) Oh goody, trig substitution!!!

$$I = 16\pi \int_0^2 \frac{dx}{(x^2+4)^2} = 16\pi \int_0^{\pi/4} \frac{2\sec^2\theta d\theta}{(4\tan^2\theta+4)^2} = 16\pi \int_0^{\pi/4} \frac{2\sec^2\theta d\theta}{16(\tan^2\theta+1)^2}$$
$$= \pi \int_0^{\pi/4} \frac{2\sec^2\theta d\theta}{(\sec^2\theta)^2} = 2\pi \int_0^{\pi/4} \frac{d\theta}{sec^2\theta}$$

   Er, actually, apologies. I suck at math. I got something very wrong, and then decided just to ask Mathematica.

$$I = 16\pi \int_0^2 \frac{dx}{(x^2+4)^2} = \frac{\pi}{4}(2+\pi) \tag{1}$$

   (b) Code follows.

```
function I = traprule(const,fun,lo,hi,correct,steps,iter)

fprintf('#steps    stepsize       Ihat           I              Err             Err/Err-1\
```

```
preverr = 1;
prevh=1;
for r=1:iter
    I = 0;
    h = (hi-lo)/steps;
    for i=1:steps
        xn = lo+(i-1)*h;
        xo = lo+i*h;
        I = I + (h/2)*(feval(fun,xn)+feval(fun,xo));
    end;
    I = const * I;
    err = abs(feval(correct)-I);
    fprintf(' %5i   %6e    %6e   %6e    %6e    %6e\n',steps,h,I,feval(correct), err, (err/p
    steps = steps*2;
    preverr = err;
    prevh = h;
end;
```

Here's the output:

```
>> traprule(16*pi,'Ifunc',0,2,'Isol',1,10)
#steps    stepsize       Ihat           I              Err            Err/Err-1
     1   2.000000e+00    3.926991e+00   4.038197e+00    1.112066e-01    5.560331e-02
     2   1.000000e+00    3.974115e+00   4.038197e+00    6.408272e-02    1.152498e+00
     4   5.000000e-01    4.021888e+00   4.038197e+00    1.630986e-02    5.090251e-01
     8   2.500000e-01    4.034110e+00   4.038197e+00    4.087396e-03    5.012179e-01
    16   1.250000e-01    4.037175e+00   4.038197e+00    1.022454e-03    5.002959e-01
    32   6.250000e-02    4.037942e+00   4.038197e+00    2.556510e-04    5.000734e-01
    64   3.125000e-02    4.038134e+00   4.038197e+00    6.391509e-05    5.000183e-01
   128   1.562500e-02    4.038181e+00   4.038197e+00    1.597892e-05    5.000046e-01
   256   7.812500e-03    4.038193e+00   4.038197e+00    3.994739e-06    5.000011e-01
   512   3.906250e-03    4.038196e+00   4.038197e+00    9.986852e-07    5.000003e-01
```

(c) The ratio seems to be approaching 2.5

(d) Code, again, follows.

```
function I = simprule(const,fun,lo,hi,correct,steps,iter)

fprintf('#steps    stepsize       Ihat           I              Err             Err/Err-1\

preverr = 1;
prevh = 1;
for r=1:iter
    I = 0;
    h = (hi-lo)/steps;
            for i=1:2:(steps-1)
        xn = lo+(i-1)*h;
        xo = lo+i*h;
        xp = lo+(i+1)*h;
        I = I + (h/3)*(feval(fun,xn)+4*feval(fun,xo)+feval(fun,xp));
    end;
    I = const * I;
    err = abs(feval(correct)-I);
    fprintf(' %5i   %6e    %6e   %6e    %6e    %6e\n',steps,h,I,feval(correct), err, (err/p
    prevh= h;
    steps = steps*2;
    preverr = err;
```

```
end;
```

Output below

```
>> simprule(16*pi,'Ifunc',0,2,'Isol',1,10)
#steps    stepsize       Ihat           I              Err            Err/Err-1
     1   2.000000e+00    0.000000e+00   4.038197e+00   4.038197e+00   2.019099e+00
     2   1.000000e+00    3.989823e+00   4.038197e+00   4.837476e-02   2.395859e-02
     4   5.000000e-01    4.037812e+00   4.038197e+00   3.855704e-04   1.594098e-02
     8   2.500000e-01    4.038184e+00   4.038197e+00   1.324213e-05   6.868849e-02
    16   1.250000e-01    4.038197e+00   4.038197e+00   8.063137e-07   1.217801e-01
    32   6.250000e-02    4.038197e+00   4.038197e+00   5.005012e-08   1.241455e-01
    64   3.125000e-02    4.038197e+00   4.038197e+00   3.122706e-09   1.247831e-01
   128   1.562500e-02    4.038197e+00   4.038197e+00   1.950839e-10   1.249455e-01
   256   7.812500e-03    4.038197e+00   4.038197e+00   1.219025e-11   1.249744e-01
   512   3.906250e-03    4.038197e+00   4.038197e+00   7.629453e-13   1.251730e-01
```

(¢)